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High-resolution superconducting single-flux quantum comparator for sub-Kelvin temperatures

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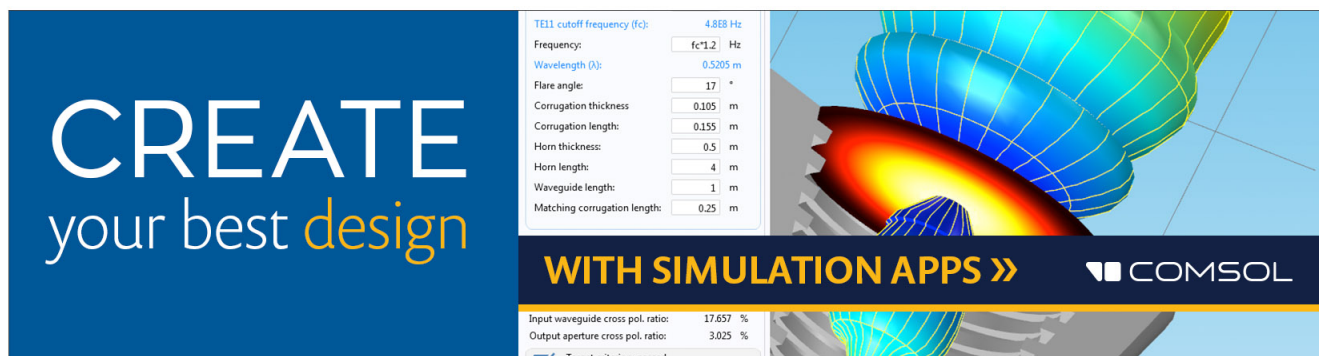
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TE11 cutoff frequency (fc):	
Frequency:	4.868 Hz
Wavelength (λ):	0.5205 m
Flare angle:	17 °
Corrugation thickness:	0.105 m
Corrugation length:	0.155 m
Horn thickness:	0.5 m
Horn length:	4 m
Waveguide length:	1 m
Matching corrugation length:	0.25 m

Input waveguide cross pol. ratio: 17.657 %
Output aperture cross pol. ratio: 3.025 %
☒ Target criterion: passed

High-resolution superconducting single-flux quantum comparator for sub-Kelvin temperatures

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A design of sub-Kelvin single-flux quantum (SFQ) circuits with reduced power dissipation and additional cooling of shunt resistors has been developed and characterized. The authors demonstrate operation of SFQ comparators with current resolution of 40 nA at 2 GHz sampling rate. Due to improved cooling the electron temperature in shunt resistors of a SFQ comparator is below 50 mK when the bath temperature is about 30 mK. © 2006 American Institute of Physics.

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Fast development of quantum information processing and its particular realization based on superconducting quantum bits (qubits) give rise to the need of development of fast low noise support electronics. Fully scalable solutions require the control circuits to be placed near qubits, i.e., at temperature far below 1 K, which significantly limits power dissipation of the control electronics. Nowadays single-flux quantum (SFQ) digital circuits¹ are considered as promising classical devices for qubit support: they are very fast and operate at low temperature and their power dissipation is substantially smaller as compared to that of semiconductor devices. In spite of rather low power dissipation and high sensitivity, SFQ logic requires significant modification for operation at sub-Kelvin temperatures. The direct rescaling of the existing SFQ circuits can be realized by the reduction of the operation frequency and critical currents of the junctions.^{2,3} The final choice of SFQ circuit parameters is determined by a compromise between required speed and possible level of power dissipation. The technological standards for niobium trilayer fabrication process set additional limitations to the circuit parameters.

Josephson junction (JJ) based balanced comparator^{4–6} is a basic element of the family of SFQ digital electronics and it can be used for testing new approaches and solutions aimed at developing new milli-Kelvin SFQ circuits. In the regime where thermal fluctuations dominate over quantum effects strong dependence on temperature of the comparator uncertainty zone (gray zone)^{2,5,6} can be utilized for measurements of the electronic temperature in SFQ circuits. Moreover, the width of the gray zone directly characterizes current (or magnetic flux) sensitivity and noise properties of SFQ circuits. In this letter we report on the results of our approach to fabricating SFQ circuits with special attention to cooling of resistors and demonstrate operation of SFQ comparators with high current resolution and low power dissipation at bath temperatures down to 30 mK.

A few important issues should be solved to satisfy the requirements for operation of SFQ circuit at bath temperatures below 100 mK. Main dissipation elements in SFQ circuits are bias and shunt resistors. The noise characteristics of the SFQ circuits are mainly determined by shunt resistors and the effect of power dissipation in bias resistors can be reduced by keeping them on a separate chip. We focus our attention to the influence of the shunt resistors and junction parameters which significantly affect operation of SFQ circuits at low temperature. The energy dissipated in a shunt resistor during one switching event is $Q \approx I_c \Phi_0$, where I_c is a critical current of the junction and $\Phi_0 = h/2e$ is the flux quantum. The power dissipation is then proportional to the operation frequency f : $P \approx f I_c \Phi_0$. As it follows from this equation reduction of both the critical current and operation frequency leads to lower dissipation in SFQ circuits. There are reasons why one should not lower these values indefinitely. High operation speed being the main advantage of SFQ logics allows one to process a reasonable number of operations during the coherent evolution of a qubit. From this point of view operation frequency should be several gigahertz or higher. Maximum operation frequency of JJ based logic should be well below the plasma frequency of the junction, which in the case of niobium trilayer process requires the critical current density to be above 10 A/cm² for $f=2$ GHz. This level is in line with the technological limits: 30 A/cm² is the lowest commercially available current density. In the situation of fixed critical current density, reduction of the JJ critical current is limited by lithography resolution and, finally, for the 30 A/cm² process, reasonable JJ critical current is of the order of a few microamperes. As a result the power dissipation in shunt resistor (assuming $I_c=2$ μ A and $f=2$ GHz) can be reduced to 8 pW.

The main bottleneck of electron cooling in the resistor is electron-phonon coupling which is relatively weak at sub-Kelvin temperatures. Electron temperature T_e and the lattice temperature T_p in the resistor are related as follows:^{7,8} $P_{e-p} = \Sigma \Lambda (T_e^5 - T_p^5)$. Here P_{e-p} is the heat flux between the electrons and the lattice, Σ is a material constant, and Λ is the

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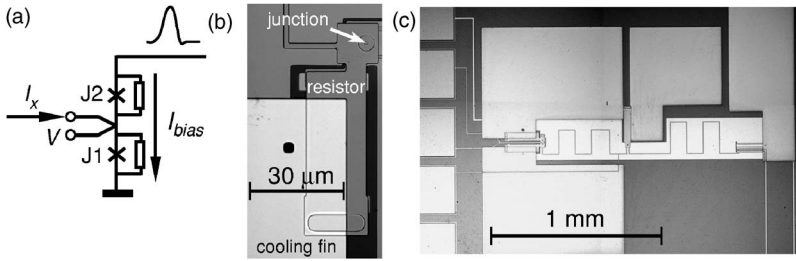


FIG. 1. Equivalent circuit of a SFQ comparator (a). Optical photograph of a shunt resistor with connection to a cooling fin in (b) and of the measured comparator with cooling fins in (c).

volume of the resistor. For metals, typical value of Σ is $\Sigma \approx 1 \times 10^9 \text{ W m}^{-3} \text{ K}^{-5}$.⁹ This indicates that electron-phonon heat transport depends on the resistor volume. It was demonstrated recently that in the case of standard resistor size ($\Lambda \approx 10 \text{ } \mu\text{m}^3$) and power dissipation of 10–20 pW, electron temperature in a shunt resistor was about 500 mK, even when bath temperature was below 100 mK.² The volume of a shunt resistor realized as a thin resistive film (with thickness of 40–100 nm) can be increased only up to 100–500 μm^3 without introducing harmful parasitic capacitance. Further cooling of electrons can be achieved by connecting to the resistors cooling fins with large volume and high thermal conductivity.^{2,3} Following these requirements Nb trilayer process with an additional 800 nm thick copper layer¹⁰ has been developed for sub-Kelvin SFQ circuits. Theoretical analysis of thermal budget of circuits with reduced critical currents and low operation frequency predicts a possibility to reduce electron temperature in shunt resistors to below 100 mK.³

A balanced SFQ comparator^{4,5} consists of two identical overdamped JJs [J1 and J2 in Fig. 1(a)] biased in series by dc I_{bias} below critical current. SFQ pulses generated by a driver (a JJ biased by constant voltage) are applied to the comparator. SFQ pulses escape from the circuit either via junction J1 or via J2 depending on the sign of the signal current I_x . Thermal and quantum fluctuations introduce uncertainty between these two processes. Detailed theoretical analyses of the gray zone for the balanced comparator have been done,^{4,11} and the theoretical model demonstrates good agreement with experiment.^{5,6} At relatively low SFQ pulse repetition rate, the dependence of switching probability (p) of one of the two junctions on signal current I_x is described by an error function and the probability density dp/dI_x is represented by a Gaussian distribution. The gray zone of the comparator ΔI_x is usually defined as^{4–6,11}

$$\Delta I_x \equiv \left| \frac{dp}{dI_x} \right|_{p=1/2}^{-1}. \quad (1)$$

Thermal noise limits sensitivity of the balanced comparator at high temperatures, while at low temperatures quantum noise starts to dominate. The crossover temperature T^* between thermal and quantum limits is determined by comparator parameters: $T^* \approx eV_c / \pi k_B$, where $V_c = I_c R_s$. According to conventional SFQ design priorities the value of shunt resistors, R_s , in SFQ circuits corresponds to the critical damping $\beta \approx 1$. Furthermore, higher resistance value is desirable for the reduction of current noise generated by shunt resistor. However, in order to scale the gray zone down with the reduction of temperature, the comparator should be in the thermal limit at the lowest bath temperature (or, more exactly, at the lowest electron temperature of shunt resistors), which may require stronger damping via the reduction of

resistance of shunts. The width ΔI_x of the “gray zone” in the regime dominated by thermal fluctuations depends on JJ critical current and temperature as (see, e.g., Refs. 4 and 11)

$$\Delta I_x \approx \alpha (2\pi I_T I_C)^{1/2}. \quad (2)$$

Here $I_T = 2\pi k_B T / \Phi_0$ and α is a dimensionless parameter of order unity determined by comparator and driver characteristics.

The comparators (Fig. 1) were fabricated using 30 A/cm² Nb trilayer process with an additional 800 nm thick Cu layer used for heat sinks of shunt resistors and with the following parameters: $I_c = 2.1 \text{ } \mu\text{A}$ and $R_s = 15.4 \text{ } \Omega$. For these parameters, the junctions are overdamped, and the crossover temperature T^* between the regimes of thermal and quantum fluctuations is 120 mK. The volume of the cooling fins is about $4 \times 10^5 \text{ } \mu\text{m}^3$. The comparator with similar parameters but without cooling fins was fabricated on the same chip in order to compare operation of the two devices. Figures 1(b) and 1(c) illustrate practical implementation of cooling fins in SFQ circuits.

The measurement procedure is based on the idea suggested in Ref. 6: the width ΔI_x of the gray zone can be obtained by the measurements of dc voltage V across one of the comparator junctions as a function of the applied current I_x . To increase the accuracy of the gray zone measurements we perform modulation measurements with a lock-in technique: the low frequency (5–20 Hz) modulation of I_x is used and the corresponding low frequency ac voltage across the comparator junction is measured. This way we measure directly the probability density dp/dI_x . The measurements were carried out at the frequency range essentially below the plasma frequency where the dependence of dp/dI_x on I_x does not deviate noticeably from Gaussian. Typical measured dc voltage across comparator junction J1 and probability distri-

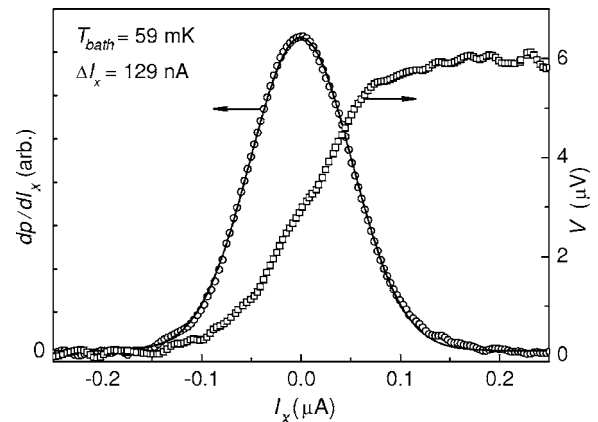


FIG. 2. Voltage across junction J1 (squares, right scale) and density of switching probability (circles, left scale) as a function of the signal current I_x . The solid line corresponds to a Gaussian fit.

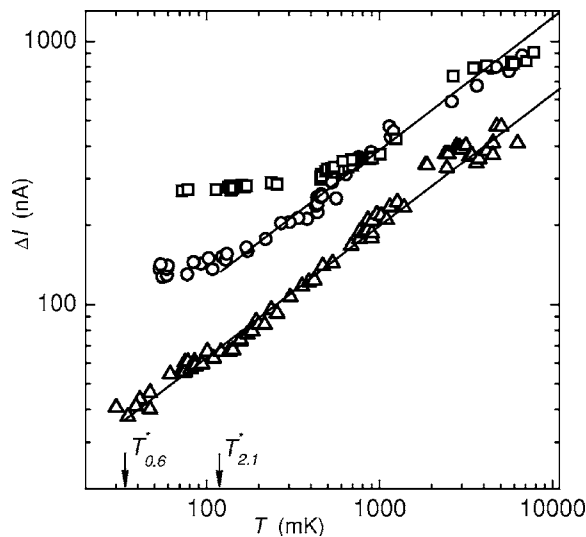


FIG. 3. Temperature dependence of the gray zone for the comparator without cooling fins [$I_c=2.1 \mu\text{A}$ (squares)] and for the comparators with cooling fins [$I_c=2.1 \mu\text{A}$ (circles), $I_c=0.6 \mu\text{A}$ (triangles)]. The solid lines represent corresponding theoretical dependences of the gray zone in the thermal limit [Eq. (2) with $\alpha=0.5$]. Crossover temperatures between thermal and quantum limits for the comparators with $I_c=2.1 \mu\text{A}$ ($T_{2.1}^*$) and $I_c=0.6 \mu\text{A}$ ($T_{0.6}^*$) are marked by arrows.

bution for the comparator with cooling fins at bath temperature of 59 mK are presented in Fig. 2. The value of the gray zone was derived from a Gaussian fit of experimental data (solid line in Fig. 2). The difference between the gray zone values calculated from fitting results and those directly from experimental data using Eq. (1) was below 5% for the data presented below. At higher operation frequencies (above 5 GHz) the measured probability distribution starts to deviate from the Gaussian form which was used to identify the upper operation frequency of the comparator.

Temperature dependence of the gray zone for the comparators without and with cooling fins operating at 3 GHz is presented in Fig. 3. The solid line corresponds to the theoretical prediction for the thermal limit [Eq. (2)], assuming that the effective electron temperature of the resistors coincides with the bath temperature T . The saturation temperatures are very different for the two comparators. Whereas the gray zone of the comparator without cooling fins saturates at the value corresponding to $T_c \approx 400$ mK, the comparator with cooling fins demonstrates further reduction of the gray zone with saturation temperature well below 200 mK. The observed saturation of the gray zone for the comparator without cooling fins is clearly above quantum limit and is caused by overheating of electrons. The saturation temperature for the comparator with cooling fins is only slightly higher than

the crossover temperature $T^*=120$ mK for this comparator, and in this case the quantum fluctuations seem to affect the final width of the gray zone.

Subsequent increase of the comparator resolution can be achieved by further reduction of the critical current. The optimal way to achieve this without degradation of frequency characteristics of the comparator is the reduction of junction size. Due to the resolution of lithography we reduced the critical current by the reduction of the critical current density down to 10 A/cm^2 . In this case the maximum operation frequency of a balanced comparator was only slightly above 2 GHz, but it was enough to demonstrate increase of sensitivity. The temperature dependence of the gray zone for the comparator with cooling fins and reduced critical current ($I_c=0.6 \mu\text{A}$, $R_s=15.4 \Omega$) is presented in Fig. 3. The increase of the comparator resolution is obtained due to both lower critical current and lower power dissipation. High current resolution and low equivalent noise temperature achieved confirm the good perspectives of SFQ circuits as control electronics for sub-Kelvin temperatures.

In summary, we presented an approach for the design of sub-Kelvin SFQ circuits with reduced power dissipation and improved cooling of shunt resistors. We have fabricated and tested a basic SFQ circuit—a balanced comparator—at bath temperatures down to 30 mK. Due to reduced power dissipation and improved cooling of the shunt resistors the effective noise temperature of SFQ devices can be reduced to below 50 mK, and the current resolution of SFQ comparators can be as high as 40 nA at 2 GHz sampling rate.

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